MULTIOBJECTIVE OPTIMIZATION IN STRUCTURAL DESIGN:
THE MODEL CHOICE PROBLEM

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Existing and potential applications of multi-optimization techniques to structural design are reviewed.

Two approaches are available to formulate a multiobjective structural design problem. The first approach starts with a classical design, say minimize weight subject to cost, reliability, risk and other constraints; and then some of the quantities included in the constraints, in particular cost and reliability, are used to define additional objectives. Thus, if X denotes the design or decision variable vector, $W(X)$, $K(X)$ and $R(X)$ the weight, cost and reliability objective functions, respectively, and $G(X)$ is a set of non-negativity constraints, the multiobjective problem is written as

Problem P1:

$$\text{Min } \underline{Z}(\underline{X}) = (W(\underline{X}), K(\underline{X}), 1-R(\underline{X})) \qquad (1)$$
$$\underline{X}$$

subject to

$$G(\underline{X}) \geq 0$$

Note that $G(\underline{X})$ usually includes constraints on the objectives themselves such as allowable maximum cost or minimum reliability.

The second approach consists in modeling the design problem directly in multiobjective form. This formulation may lead not only to problem P1, but also to the inclusion of qualitative objectives into the analysis, expressed by criteria such as aesthetics $A(\underline{X})$ and employment $M(\underline{X})$. Since qualitative (ordinal) objectives are usually defined on a discrete scale, it is convenient to consider a discrete set of alternatives as well. Accordingly, let $\underline{X} = \{\underline{X}(i): i=1,2,\ldots, J\}$ be a discrete set of alternative designs. Then the jth alternative design is evaluated by the criterion vector

$$\underline{C}(j) = (W(j), K(j), 1-R(j), A(j), M(j))$$

and the multiobjective problem now becomes:

Problem P2:

Find an alternative $\underline{X}(j)$ that constitutes a satisfactory trade off between the elements of criterion vector $\underline{C}(j)$.

For example, consider the standard steel-floor design (1) as described in (2), in which cost is to be minimized. The optimization technique used in that model is geometric programming (3), (4). Alternatives may be obtained by minimizing weight, or by probabilistic design (5), (6). Table 1 shows five alternatives obtained from the basic model of (2). Design I represents the original problem, Design IV corresponds to a minimum weight formulation, Designs II, III, V are probabilistic. These alternatives have been obtained by changing constraints into objectives, which means that Table 1 stems from Problem P1.

TABLE I Alternative Steel Floor Designs Versus Four Objectives

	DESIGN				
	I	II	III	IV	V
Cost	3850	3085	2774	4780	4162
V-Ratio	1:1	1:2	1:2	1:1	1:2
Reliability	1.00	0.80	0.95	1.00	.95
Applied Weight	1000	720	570	1000	570

Design I   Standard (Deterministic)

Design II   Probabilistic, R = .28

Design III Probabilistic, R = .43

Design IV   Minimum U_T

Design V   Minimum U_T and Probabilistic R = .43

For both problems P1 and P2, a trade-off solution, also called "satisfactum", is to be sought among the set of non-dominated solutions or Pareto-optimum set. Alternative k dominates alternative j if $\underline{C}^*(k) \geq \underline{C}(j)$; the dominance is strict if at least one element of vector $\underline{C}(k)$ is greater than the corresponding element of $\underline{C}(j)$.

Once the multiobjective problem has been formulated as either problem P1 or P2, a solution technique which matches with the type of problem and desiratas of the decision-maker is to be chosen. This model choice problem is examined in a systematic manner and illustrated by setting up a problem with a choice between eleven multiobjective techniques, respectively:

1. Compromise Programming (7), (8)
2. Goal Programming (9), (10)
3. Cooperative Game Theory (11), (12)
4. Multiattribute Utility Theory (13), (14)
5. Surrogate Worth Trade-off (15)
6. ELECTRE (16), (17), (18)
7. Q-analysis (19), (20)
8. Dynamic Compromise Programming (21), (22), (23)
9. PROTRADE (24), (25)
10. STEP Method (26)
11. Local Multiattribute Utility Functions (27).

These techniques can be categorized by means of five binary classification criteria:

a. Marginal versus non-marginal difference between alternatives; are only marginal differences between alternatives being considered? If yes, formulation P1 is applicable; if not, that is, if major differences between alternatives are possible, say an arch versus a gravity dam, then formulation P2 may be preferable. A parallel classification criterion would be design versus maintenance problem.

b. Qualitative versus quantitative criteria: are there qualitative criteria which cannot or should not

be quantified? If so, formulation P2 may be more appropriate than formulation P1.

c. Prior versus progressive articulation of preferences: at which point of the analysis is the decision-maker required to express his preference function, if at all?

d. Interactive versus non-interactive: has the technique been explicitly designed for an interactive mode of application?

e. Comparison of alternatives to a given solution point or to each other; in the former case, the solution point may be an aspiration level, corresponding to a feasible solution, or a goal point, corresponding to a non-feasible (often ideal) solution.

To these five classification criteria are added other criteria describing the characteristics of the problem (size, uncertainty, number of objectives...), the decision maker (level of understanding, time available for interaction) and the techniques themselves (robustness, partial versus complete ranking provided, ease of use...). This procedure leads to defining four categories of choice criteria (23):

1. mandatory binary criteria: for example, under formulation P1, a technique able to solve only discrete problems would be eliminated from further consideration

2. non-mandatory binary criteria: for example, comparison to an aspiration level versus comparison of a goal point

3. technique-dependent criteria: time required from decision-maker, robustness

4. application-dependent criteria: number of objectives, formulation P1 or P2.

To conclude, the advantages of a multiobjective formulation over a single objective one with a sensitivity analysis is that more alternatives can be explored and that explicit trade-offs between criteria can be made. Furthermore, given any problem involving trade-offs between quantitative or even qualitative criteria, an appropriate multiobjective technique can usually be found by following the proposed model choice procedure. The potential use of multiobjective techniques in structural design thus looks quite promising.

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